$(\frac{7}{2},\frac{7}{2})$ masses within our scheme. Assuming the pion and nucleon masses and the πNN coupling constant to be known, these turn out to be 1200, 1640, and 2310 MeV. It is a curious fact that, together with the nucleon, which has a mass of 940 MeV, these masses m_J obey to a few percent the rigid rotator formula $m_J = AJ(J+1)+B$, where A and B are constants. This is exactly the prediction of the strong-coupling model[16] which, however, had an additional arbitrary parameter.

The above masses of the $(\frac{3}{2},\frac{3}{2})$ and $(\frac{5}{2},\frac{5}{2})$ particles should be compared with the experimental values of 1240 and 1560 MeV, respectively. In the latter case we are assuming, of course, that we can identify our particle with the resonance of Ref. 7. [Actually, the value $\omega_{5/2,5/2}$ in Eq. (2.14) does not coincide with the maximum of the cross section; it corresponds to 1650 MeV, which may be the more appropriate quantity to compare with our calculated value.]

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Impact-Parameter K-Matrix Approach to High-Energy Peripheral Interactions*

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An approximate dispersion-theoretic treatment of peripheral inelastic processes is introduced with the aid of a K-matrix formalism based on the impact-parameter representation of Blankenbecler and Goldberger. The method allows the use of one-meson exchange poles as a framework for constructing a multichannel scattering amplitude which satisfies unitarity in the high-energy region, allowing for an indefinitely large number of open channels. The reaction matrix is time-reversal symmetric and exhibits any other symmetries of the pole terms. Applications are numerically worked out for models of high-energy $\bar{K}p$ and np charge exchange, and in the former case satisfactory agreement with experiments is achieved. A qualitative discussion is given of peripheral isobar production models. The high-energy $\bar{p}p$ and $\bar{K}p$ diffraction scattering is examined, as well as the agreement of the small-momentum-transfer behavior with a simple model not involving Regge poles. The method sheds no light on the difference between $\bar{p}p$ and pp scattering at high energies.

I. INTRODUCTION

ELASTIC and inelastic reaction amplitudes of elementary particles and isobars at high energies characteristically exhibit a peak in the forward direction. In some reactions, such as proton-antiproton elastic scattering,[1] the form of the amplitude can be readily interpreted by analogy with optical diffraction patterns, suggesting a semiclassical picture of the nucleon with an absorptive core and a diffuse boundary, phenomenologically of Gaussian shape. In some other cases, for example[2] $K^{+}+p \to K^{0}+N_{3/2}^{*++}$, the center-of-mass angular distribution of the production reaction is clearly consistent with a one-meson exchange formula. The most common high-energy reaction behavior seems to be intermediate between these extremes.

Phenomenological corrections to one-particle exchange formulas based on the introduction of form factors have been widely used in the analysis of peripheral inelastic processes,[3] but these form factors have at least two objectionable properties. The first is lack of generalizability; evidence has accumulated that such a form factor appropriate to the vertex $\rho\pi\pi$ has a behavior much different from that for the ρKK vertex,[2] while a close relation between these form factors would be expected in various symmetry schemes such as unitary symmetry.

The second is a lack of theoretical foundation within the framework of dispersion, or on-the-mass-shell, techniques. A form factor may be expected to have an important influence in a perturbation-theoretic approach, but even then it is difficult to see the source of such large variations as are required to fit the data. This point has been discussed by Durand and Chiu,[4] Ross and Shaw,[5] and earlier by Baker and Blankenbecler.[6]

The authors (particularly Refs. 4 and 5) also point out that the inclusion of initial and final-state interactions, usually taken to be strong elastic scattering with a diffraction character, is very important in the analysis of peripheral inelastic processes; and, in fact, these corrections may be quite sufficient to explain the deviations from one-meson exchange previously ascribed to form factors. Essentially the same conclusion has been reached by Dar and Tobocman in a slightly different language; a detailed discussion of the mechanism has been given by Gottfried and Jackson.[7]

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[1] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 503 (1963).
[2] V. Barger and E. McCliment, Phys. Letters **9**, 191 (1964).
[3] E. Ferrari, Nuovo Cimento **30**, 240 (1963); E. Ferrari and F. Sellari, Nuovo Cimento **27**, 1450 (1963).
[4] L. Durand and Y. T. Chiu, Phys. Rev. Letters **12**, 399 (1964).
[5] M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 672 (1964).
[6] M. Baker and R. Blankenbecler, Phys. Rev. **128**, 415 (1962).
[7] A. Dar and W. Tobocman, Phys. Rev. Letters **12**, 511 (1964); A. Dar, *ibid.* **13**, 91 (1964). K. Gottfried and J. D. Jackson, CERN paper, 1964 (unpublished).

The investigations mentioned above share, however, certain drawbacks which impede a theoretical understanding of the reactions. They assume that the inelasticity is due to a fictitious absorptive potential, or directly introduce a phenomenological complex phase shift for the elastic scattering amplitudes. This makes an intuitive grasp of the physics involved difficult, and one has no basis for generalizing from one reaction to another.

The impact-parameter K-matrix formalism to be presented below has the advantages of the approaches used by the preceding authors, with additional flexibility and a more direct contact with physical models. It will be possible to construct a set of scattering and reaction amplitudes which is based entirely on one-meson exchange processes, if desired. The set of amplitudes will in any case satisfy multichannel unitarity in a high-energy limit. The imaginary parts of the amplitudes will be nonvanishing, and polarization effects may be easily handled. Time reversal symmetry will be automatically satisfied, as well as any other higher symmetries one wishes to introduce into the meson-exchange pole terms.

Theoretical justification for the formulas are supplied only insofar as the behavior for small momentum transfer is treated. A complete theory including large momentum transfer behavior is not attempted, although the formalism allows a phenomenological treatment of short-range reactions to be introduced in a unitary way.

After the basic approximations and formalism have been set down, simple models will be quantitatively examined for $\bar{p}p$ elastic scattering, $\bar{K}p$ charge exchange, and np charge exchange. In the latter two cases it is found that the available energy data have a plausible interpretation in terms of a meson exchange model. The $\bar{p}p$ diffraction can be fit by a more complicated model. A qualitative examination of the vector-meson exchange model for isobar production then shows that, contrary to the conclusions of Ref. 2, the model is probably quite adequate to explain the available data without form factors when the unitary modifications are included as in the analysis of the present work.

II. BASIC FORMALISM AND APPROXIMATION

The object of this work is the development of a multichannel, unitary representation for reactions at high energies. Although we could treat many-particle states, it will be much simpler to begin with a representation of the effects of inelastic channels by including only two-body open channels. One surmises that the unitarity effects of multiparticle states on selected two-body reactions may be simulated in this way, if isobars are included among the final state objects.

As a second simplification, we assume that there are a large number of open channels below the energy we are interested in, and that any nearby thresholds are relatively unimportant. Presumably, the latter would be true if we actually were considering multiparticle states from a statistical viewpoint, since the final state many-body phase space would be small close to threshold for any particular channel. Together with this, we assume the momenta in all channels are large compared to the masses, so we can use high-energy limit conditions in all the open channels (not only the lowest, elastic scattering channel).

We will ignore spin in the initial formulation, although this can easily be incorporated if more detailed properties of the reactions are to be computed. In the examples treated later, one of the initial particles is always a proton, but we will ignore effects connected with its spin in developing the formalism.

A further simplification of the problem is necessary to reduce the algebra involved. We will take most of the inelastic final states to be noninteracting, and allow transitions only to and from the elastic scattering channel. This will still allow us to accomplish the objective outlined in the introduction; if the effects of inelasticity enter as an incoherent sum over a large number of channels, we suppose that the interactions in any one channel have relatively little influence on the sum. It is clear that such an approach is tenable only at high energies. We will also ignore any possible effects of anomalous thresholds or complex singularities, since we are in a very high-energy region.

Finally, baryon exchange terms will be ignored, which corresponds to using poles only in the momentum transfer variable. This is appropriate for meson-exchange models.

The formulation of scattering amplitudes will now be taken in the Fourier-Bessel representation developed for dispersion theory by Blankenbecler and Goldberger (BG),[8] and further amplified by Baker and Blankenbecler (BB).[6] We could alternatively develop the formalism in the partial-wave series representation, but the impact-parameter picture which accompanies the Fourier-Bessel representation aids physical understanding of the approximations involved and is readily adapted to phenomenological approximations for the large momentum transfer behavior. In addition, the formalism has been discussed at some length in BG and BB, and we will merely summarize the relevant formulas up to the point where our approximation methods become significantly different from those of BB.

Following the covariant normalization of BG, Sec. VII, we define the multichannel, two-body scattering matrix $\mathbf{M}$ such that the center-of-mass elastic scattering differential cross section for scattering of particle 1 on a proton (11 channel) is given by

$$\frac{d\sigma}{d\Omega}=|M_{11}(s,t)|^2\times\frac{(M_p+M_1)^2}{s}, \tag{2.1}$$

[8] R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962).

where s is the square of the total c.m. energy, M_p is the proton mass, and M_1 the mass of particle 1. The M_{ij} have singularities only for positive s and positive t under our approximations.

The Fourier-Bessel components of $\mathbf{M}$ will be denoted by $\mathbf{H}$;

$$\mathbf{M}(s,t)=\int_0^\infty bdb\, J_0(b(-t)^{1/2})\mathbf{H}(s,b^2)\,, \qquad (2.2)$$

and we do not introduce the singularities in u, which lead to two signatures as in Sec. VII of BG, since we are ignoring baryon exchange singularities.

In the limit $k_ib\gg 1$ for all i, where k_i is the center-of-mass momentum in channel i, the matrix $\mathbf{H}$ now should be taken to satisfy the multichannel unitarity condition in the form [compare BG, Eq. (7.11) and Sec. III]

$$\text{Im}\mathbf{H}=\mathbf{H}^\dagger\mathbf{r}(s)\mathbf{H}\,, \qquad (2.3)$$

where $\mathbf{r}$ is a diagonal matrix of phase space factors appropriate to the FB representation: Explicity, we put

$$r_{ii}(s)=(M_{1i}+M_{2i})/(2k_is^{1/2})\,, \qquad (2.4)$$

where M_{1i}, M_{2i} are the masses of the two bodies in the ith channel. We will later assume that most of the important channels have particles of about the same mass, and use $2M_{\text{av}}$ instead of $(M_{1i}+M_{2i})$; furthermore we will replace k_i by $s^{1/2}/2$, since the energy is large compared to the masses. It is easy to keep the more general form for $\mathbf{r}$ however at this point. Now, if we write

$$\mathbf{H}(s,b^2)=\mathbf{K}(s,b)[\mathbf{I}-i\mathbf{r}(s)\mathbf{K}(s,b)]^{-1}\,, \qquad (2.5)$$

with a real matrix $\mathbf{K}$, the function $\mathbf{H}$ will automatically satisfy asymptotic unitarity condition (2.3). This representation will form the basis for our approximation methods.

Before explicitly introducing meson-exchange models, we will comment on the relation between (2.5) and other unitary calculation methods. It was pointed out in BG (Eq. 3.20) that the large-b Fourier-Bessel components of the scattering amplitude are proportional to the partial-wave amplitudes for large l, if b and l are related by

$$kb=l+\tfrac{1}{2}\,. \qquad (2.6)$$

The condition $kb\gg 1$ is required for this association, but that is precisely the condition under which (2.5) is automatically unitary for real $\mathbf{K}$. Compare the partial-wave representation of the elastic scattering matrix element

$$M_{11}(s,t)=\frac{s}{(M_p+M_1)}\sum_J(2J+1)\times P_J(\cos\theta)\frac{e^{2i\delta_J}-1}{2ik_1}\,, \qquad (2.7)$$

with the representation (2.5) as follows: Writing $e^{2i\delta_J}-1/i=\tau_J/1-i\tau_J$, where $\tau_J(s)=\tan\delta_J(s)$, we can pass to the limit of a large number of angular momentum contributions and write

$$M_{11}(s,t)=\frac{s^{1/2}k_1}{(M_p+M_1)}\int_0^\infty\frac{l+\frac{1}{2}}{k}\times d\left(\frac{l+\frac{1}{2}}{k}\right)P_l(\cos\theta)\frac{\tau_l(s)}{1-i\tau_l(s)}\,.$$

Then employing the asymptotic representation for large l

$$P_l(\cos\theta)\cong J_0(2(l+\tfrac{1}{2})\sin^2(\theta/2))\,,$$

we obtain by a change of variable

$$M_{11}(s,t)=\frac{s^{1/2}k_1}{(M_p+M_1)}\int_0^\infty bdb\, J_0(b(-t)^{1/2})\frac{\tau(b,s)}{1-i\tau(b,s)}\,,$$

where $b=(l+1/2)/k$, and $\tau(b,s)=\tau_l(s)$ with this identification. It is clear therefore that if only a single elastic channel is open, we can write

$$\frac{s^{1/2}k_1}{(M_p+M_1)}\tau(b,s)=K_{11}(s,b)\,. \qquad (2.8)$$

In general, when a number of inelastic channels are considered, we will obtain a complex phase shift, since $\mathbf{K}$ is to remain real; the relation between the K-matrix elements and τ involving the off-diagonal terms will contribute an extra imaginary part to $\tau(b,s)$. This will be the source of the absorptive part of the phase shift, heretofore treated in a purely phenomenological way.

Our further approximations will be based on a use of the first Born approximations, or meson-exchange pole terms, for the K-matrix elements. These are

$$K_{ij}^{(B)}=H_{ij}^B(s,b^2)=\int_0^\infty xdx\, J_0(bx)M_{ij}^B(s,-x^2)\,, \qquad (2.9)$$

where $M_{ij}^B(s,t)$ are the pole approximations for the reaction amplitude $i\to j$; these may be scalar or pseudoscalar-meson exchange or vector-meson terms, the latter written as Regge poles if the high-energy behavior is important. It was proved in BG that the one-channel elastic amplitude in a nonrelativistic Yukawa potential scattering model was well approximated in the limit $kb\gg 1$ by

$$H_{11}(s,b^2)=\frac{H_{11}^B}{1-iH_{11}^B/2k} \qquad (2.10)$$

(Eq. 3.11 of BG) which corresponds to using $K_{11}^{(B)}$ in our general Eq. (2.5), ignoring inelastic channels, and using the nonrelativistic limit of the phase space function.

We conjecture that the pole terms for $\mathbf{K}$ will be good approximations in the limit when all $k_ib\gg 1$, even if we generalize to relativistic reactions and Regge poles as well as scalar meson exchanges. This must hold if unsubtracted N/D calculations done at low energies can be extrapolated to multichannel problems at high energy; if the angular momentum representation is used, we find[9]

$$\mathbf{K}_l = \mathbf{N}_l[\mathrm{Re}\mathbf{D}_l]^{-1}, \qquad (2.11)$$

and in meson-exchange models $\mathrm{Re}D \to 1$ while N approaches the pole approximation at large energies[10] or large l.

Note that such an approximation is usually much better than just taking the pole approximation for M_{ij}, since it is explicitly unitary and will give nontrivial polarization and interference phenomena.

The pole approximations for K_{ij} diverge at small b values. Typically we have

$$K_{ij}^{(B)} = R_{ij}(s)\int_0^\infty \frac{x dx}{\mu^2+x^2} J_0(bx) = R_{ij}(s) K_0(\mu b) \qquad (2.12)$$

for the exchange of a meson of mass μ, where K_0 is a modified Bessel function, with the properties

$$K_0(z) \to \gamma + \ln(2/z) \quad \text{as} \quad z \to 0,$$
$$\text{where } \gamma \text{ is Euler's constant}; \qquad (2.13)$$
$$K_0(z) \to (\pi/2z)^{1/2} e^{-z} \quad \text{as} \quad z \to \infty.$$

Although the divergence for small b is in a region where the approximations are invalid, it assures that the scattering and reaction Fourier-Bessel amplitudes have a characteristic complete "absorption" region with a diffuse boundary and some minimum radius determined by the strength and range of the interactions. In a somewhat more realistic model, the sum of many strong short-range contributions to the K matrix will produce such a "black" region for the elastic scattering amplitudes; as will be shown later, this becomes a "white" region for any particular inelastic channel.

With the assumption that the inelastic final-state particles have no interactions other than a transition back to the (11) channel, we can write the K matrix for the system in the form[11]

$$\mathbf{K}(s,b) = \begin{bmatrix} \alpha & C_2 & \cdots & C_n \\ C_2^* & 0 & \cdots & 0 \\ \vdots & \vdots & & \\ C_n^* & 0 & \cdots & 0 \end{bmatrix}. \qquad (2.14)$$

We do not yet make the approximation $K_{ij} = K_{ij}^{(B)}$, but we take the high-energy limit such that for all i, $r_{ii}(s) \approx \rho(s) = 2M/s$, where M is an average mass of particles in the channels under consideration; we take it to be around the proton mass. Now $\mathbf{H}(s,b)$ can be explicitly computed from Eq. (2.5), and we find that the elastic scattering term is

$$H_{11}(s,b^2) = \frac{\alpha + i\rho \sum_n C_n^* C_n}{1 - i\rho\alpha + \rho^2 \sum_n C_n^* C_n}, \qquad (2.15)$$

while the production terms are

$$H_{ij}(s,b^2) = \frac{C_i}{1 - i\rho\alpha + \rho^2 \sum_n C_n^* C_n}, \qquad (2.16)$$

where $\rho(s) = 2M/s$.

Our motivation now is principally to see the modification of one-meson exchange terms by the peripheral inelastic processes. We expect that all reactions couple to inelastic one-pion exchange terms, which have a b dependence characteristic of Eq. (2.12), with μ taken as the pion mass. If we assume that these are the only important contributions for large impact parameter, we can factor out the function K_0 from each of the off-diagonal elements C_j in $\mathbf{K}$, leaving a function of s only. We will determine any desired s dependence by appeal to experiment. Writing

$$C_j(s,b) = F_j(s) K_0(\mu b), \qquad (2.17)$$

with F_j real, and putting

$$G(s) = \rho^2(s) \sum_j F_j^2(s), \qquad (2.18)$$

we have the more explicit forms of the elastic scattering and production amplitudes;

$$H_{11}(s,b^2) = \frac{\alpha(s,b) + i(s/2M) K_0^2(\mu b) G(s)}{1 - i(s/2M)^{-1}\alpha(s,b) + K_0^2(\mu b) G(s)}, \qquad (2.19)$$

$$H_{1j}(s,b^2) = \frac{C_j(s,b)}{1 - i(2M/s)\alpha(s,b) + K_0^2(\mu b) G(s)}. \qquad (2.20)$$

We expect these then to give the essential corrections for large b, which means we should be able to compare the behavior of $M_{11}(s,t)$ for small t at any given energy, given $G(s)$ from experiment and α from a theory such as one-meson exchange. This prospect will be examined quantitatively for scattering in Sec. IV below.

The formula (2.20) for a single-production amplitude clearly exhibits the general qualitative behavior of our approximate amplitudes. The small-b region is damped out due to the extra factor of $K_0(\mu b)$ in the denominator. This will invariably yield a matrix element M_{1j} which is more sharply peaked in the forward direction than the uncorrected one-pion pole term. We will notice this effect in the analysis of np charge exchange. This behavior is even more pronounced, however, if we examine processes which are forbidden to occur through one-

[9] R. H. Dalitz, Rev. Mod. Phys. **33**, 471 (1961), Sec. IV.
[10] See, e.g., R. C. Arnold, Phys. Rev. **134**, B1380 (1964), Appendix.
[11] A similar form has been used by D. S. Chernavskii, Zh. Eksperim. i Teor. Fiz. **45**, 1558 (1963) [English transl.: Soviet Phys.—JETP **18**, 1072 (1964)].

pion exchange, so the longest-range inelastic K-matrix terms then are obtained from ρ or K^* meson poles. In such cases, the suppression from competing one-pion inelastic processes is large even in the forward direction. Such a case is encountered in the charge-exchange reaction $K^- + p \rightarrow \bar{K}_0 + n$, where ρ exchange presumably is the most peripheral contribution. This case will be quantitatively treated in Sec. V.

A final qualitative observation needs to be added here concerning unitarity and one-meson pole terms. In many cases involving spin, the basic pole terms obtained from field theory contain powers of t in the numerator as well as the denominator. These generate exceptional S-wave (and possibly P-wave) contributions which are not of the form obtained by analytic continuation down from high J values. There is some ambiguity in a dispersion-theoretic approach as to whether or not these exceptional low partial-wave contributions should be included in the pole approximations; the question may be resolved if one accepts the treatment of the exchanged mesons as Regge poles with a trajectory having small, but nonzero, slope. In such a case the partial-wave amplitude for $J=0$ is an analytic continuation from higher J values; the pole terms are numerically close to the field-theory results for low energies, but at high energies resemble the amplitudes with deleted S-wave terms.

The ambiguity in these low partial waves is relatively unimportant, however, when a unitary formalism [such as (2.5) above] is applied to compute the scattering amplitude. Then the effects of inelastic processes completely dominate the small-b region of $\mathbf{H}(s,b^2)$; the off-diagonal elements will vanish for small b and the diagonal terms approach some nonzero constant [cf. Eqs. (2.19), (2.20)]. This has the effect of eliminating any exceptional low partial wave terms in the pole approximations.[12] As a consequence, charge-exchange reactions proceeding through one-pion exchange do not vanish in the forward direction, and the formulae for vector meson models of isobar production[2,13] may even be qualitatively quite misleading, unless all powers of t in the numerators of the matrix elements are eliminated, e.g., by expansion in partial fractions. We will return to this discussion in Sec. VII.

The formulas (2.15) and (2.16) are closely related to the expressions (1.7) and (1.13) of BB; however, they differ in detail because of the nature of the approximations made here for the inelastic channels.

[12] This point has been discussed in Ref. 4, footnote 7. See also Fig. 4 of G. Goldhaber, W. Chinowsky, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Letters **6**, 62 (1963) for further indication of such an effect. Evaluating Δ^2 (in the numerator) on the mass shell in the cross-section formula does not give precisely the same results as in the pole-term matrix element; the unitarity conditions for various spin amplitudes must be studied when the final-state particles can have states of high spin.

[13] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters **11**, 90 (1963).

III. RELATIONSHIP TO DWBA AND OPTICAL MODELS

Motivation similar to that of our work here has led previous authors[4,5] to utilize the distorted-wave Born approximation (DWBA) for relativistic, absorptive potentials acting in the initial and final states. This leads to a formula for the partial-wave inelastic amplitude of the form[4]

$$M_{12}{}^J(s)=\exp[i\delta_J{}^{(1)}(s)]B_{12}{}^J(s)\exp[i\delta_J{}^{(2)}(s)], \quad (3.1)$$

where $B_{12}{}^J$ is the partial-wave Born approximation for the reaction, and $\delta_J{}^{(1)}$, $\delta_J{}^{(2)}$ are the complex phase shifts describing elastic scattering in channels 1 and 2, respectively. This has considerable intuitive appeal, but the introduction of an absorptive potential to simulate inelasticity is undesirable for two reasons. First, the connection between different reactions is obscure and generalizations can be made only on purely phenomenological grounds. Second, there is no provision for characteristics of the production amplitude to be reflected back into the elastic scattering channel, as must happen if unitarity is to be satisfied.

The K-matrix formalism (2.5) together with approximations using meson pole terms as in the preceding section rectifies these difficulties, although it is expected to do so accurately only for large impact parameters.

It is instructive to reproduce the result (3.1) as a limiting case of the K-matrix formulas. We simplify, however, by taking only a two-channel problem, so the phase shifts become real in the limit of small channel couplings (small off-diagonal pole terms). Then we will find (3.1) is indeed true in the limit $K_{12}=K_{21}{}^*\rightarrow 0$, or sufficiently high energy.

For a two-channel problem we would write

$$\mathbf{K}=\begin{bmatrix}\alpha_1 & \beta\\ \beta^* & \alpha_2\end{bmatrix}; \quad (3.2)$$

then in the high-energy limit, applying (2.5) yields

$$\mathbf{K}[\mathbf{I}-i\rho\mathbf{K}]^{-1}=\frac{1}{\Delta}\times\begin{bmatrix}\alpha_1(1-i\rho\alpha_2)+i\rho|\beta|^2 & \beta\\ \beta^* & \alpha_2(1-i\rho\alpha_1)+i\rho|\beta|^2\end{bmatrix}, \quad (3.3)$$

where $\Delta=(1-i\rho\alpha_1)(1-i\rho\alpha_2)+\rho^2|\beta|^2$; thus the off-diagonal matrix element is

$$H_{12}(s,b^2)=\beta/\Delta. \quad (3.4)$$

Now, for large energies, $\rho^2\rightarrow 0$; so, if at a given energy $|\beta|^2$ is negligible compared to unity—or (if β does not grow with energy) at any sufficiently high energy—this reduces to

$$H_{12}(s,b^2)\approx\frac{\beta}{(1-i\rho\alpha_1)(1-i\rho\alpha_2)}. \quad (3.5)$$

But the factors in the denominator now just give the phase of the amplitudes in the elastic scattering channels, so using (2.6) we reproduce (3.1) under the stated conditions.

It is not clear whether a complete justification of (3.1) in the limit of small off-diagonal Born terms within the framework of our model can be achieved, since there is some conflict between the complex-potential viewpoint and the representation (2.5).

Another approximation method, which has been applied by Serber[14] to high-energy proton-proton diffraction scattering, is based directly on the eikonal method[15] used with great success in studies of the nuclear optical-model potential. With rather drastic assumptions on the validity of a complex potential model, the formulas nevertheless reproduce the experimental data for large momentum transfer amazingly well. It would clearly be desirable to include this approach in the present work, but there are two obstacles to this. First, there is no justification in terms of a dispersion-theoretic foundation; in fact, this method was discarded by Blankenbecler and Goldberger in the beginning in favor of the more conservative N/D representation for just this reason. Second, and more serious from the point of view of the present work, one needs to specify in advance the absorptive potential; and it is precisely this concept that the K-matrix formalism is designed to avoid. There is clearly room for improvement in connecting the two points of view.

IV. ANALYSIS OF $\bar{p}p$ AND $\bar{K}p$ ELASTIC SCATTERING

In this section, we attempt to fit the small-angle experimental $\bar{p}p$ and $\bar{K}p$ elastic scattering cross sections with simple models for the peripheral inelastic processes. Comments on Regge poles will be reserved to the end of this section.

To begin, we shall assume the diagonal K-matrix elements K_{11}, K_{22}, etc., corresponding to the elastic scattering channels are negligible, so that all the observed cross section will be due to inelasticity. This is not quite consistent with the experimental data,[1] since extrapolation of the elastic amplitudes to zero angle and application of the optical theorem in $\bar{p}p$ scattering indicates a few percent, real part exists in the amplitude; but we shall use the assumption as a plausible first approximation.

Assuming only pion poles in the off-diagonal K-matrix elements, we are led to Eq. (2.19) with $\alpha=0$, for the elastic scattering amplitude. Now, we observe from experiment that the diffraction peaks do not change shape with energy. This leads us to put

$$G(s)=a\,, \tag{4.1}$$

where a is a constant for each reaction. Then we have

$$H_{11}(s,b^2)=i(s/2M)\frac{aK_0^2(\mu b)}{1+aK_0^2(\mu b)}\,. \tag{4.2}$$

With this formula, we are able to fit the $\bar{p}p$ data for very small momentum transfers $(-t)^{1/2}<2\mu(-t<0.08$ $\text{BeV}^2/c^2)$ including the magnitude of the elastic cross section, by choosing $a=1.8$ at 10 BeV.

For $\bar{K}p$, the smaller experimental slope and magnitude of the diffraction peak is indicative of shorter range inelastic processes. This amplitude was fit by assuming that the dominant inelastic K-matrix terms could be represented by the exchange of a heavier meson. A mass of twice the pion mass was chosen, leading to a formula for H_{11} which is obtained from (4.2) by replacing μ by 2μ, and taking M to be the average of M_K and M_p. In this case, the best fit was obtained with $a=0.6$. This fit may be regarded as phenomenological, with 3 parameters (a, M, and meson mass 2μ). However, we have set M to its most naive value, and have not chosen an effective meson mass of less than 2μ, because no physical states in the crossed channel have a mass between μ and 2μ. The fit could be considerably improved if we were to regard M and the meson mass as completely free parameters, for example if we used $1.5\,\mu$ insteady of 2μ, and increased M to a larger value.

Some improvement in the $\bar{p}p$ fits could be achieved by increasing M, which we took equal to the proton mass in the $\bar{p}p$ fit. An average mass of the inelastic reaction channels would presumably be somewhat larger than M_p in reality. The fit for $\bar{p}p$ scattering around 10 BeV is shown in Fig. 1. The behavior of H_{11} as a func-

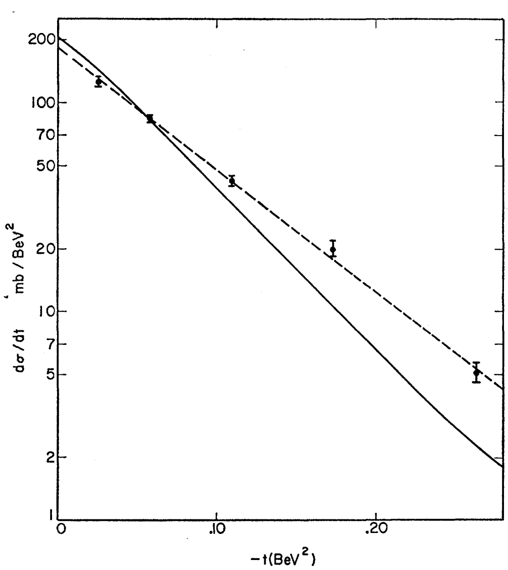

FIG. 1. Fit to $\bar{p}p$ elastic scattering at 10 BeV for small momentum transfer. Experimental points are a sample from Ref. 1; dashed line is an exponential fit to data. Solid line shows prediction of Eq. (4.2) with $a=1.8$.

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[15] R. J. Glauber, in *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959) Vol. I.

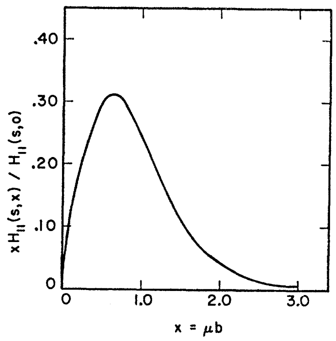

FIG. 2. Graph of $xH_{11}(s,x)/H_{11}(s,0)$ where $x=\mu b$, with H_{11} from Eq. (4.2); parameter a as in Fig. 1 for $\bar{p}p$ scattering.

tion of b for $\bar{p}p$ scattering is sketched in Fig. 2, where we have plotted $bH_{11}(s,b^2)/H_{11}(s,0)$ for the same value of a. The fit for $\bar{K}p$ elastic scattering at similar energies is shown in Fig. 3.

The $\bar{p}p$ fit is adequate for small $(-t)$; the $\bar{K}p$ fit may not be as good if smaller $(-t)$ data values continue the apparent exponential trend. We should emphasize that the normalization (for the elastic cross section at least) is also determined by our parameter a. The ratio of elastic to total cross section is also predicted by the model, by application of the optical theorem; however, this is not expected to come out well, since the total cross section depends on the small-b region which is not treated correctly.

It is clear that by adding a sufficiently large number of heavy-meson exchange poles one could approximate the experimental shape, an exponential in $(-t)$, corresponding to a Gaussian distribution in b for the large-b regions. The data are well fit by a Gaussian distribution for $H(b^2)$, but there is no physical model leading directly to such a distribution which is based on inelastic processes. The Regge pole dominance hypothesis for high-energy interactions[16] does predict an exponential drop-off in t, but if we are to fit the $\bar{p}p$ scattering with only Regge poles, it seems necessary to take a Pomeranchuk trajectory which has zero slope. If this is true, it is difficult to account for the $\bar{p}p$ scattering behavior at high energies unless the Regge pole residue functions of some vector mesons change sign[17] around $t=0$. This cannot happen in a single-channel two-body potential-theory model,[18] from which most of the Regge pole knowledge is obtained; however, the question remains open when many-particle calculations of Regge poles are considered.

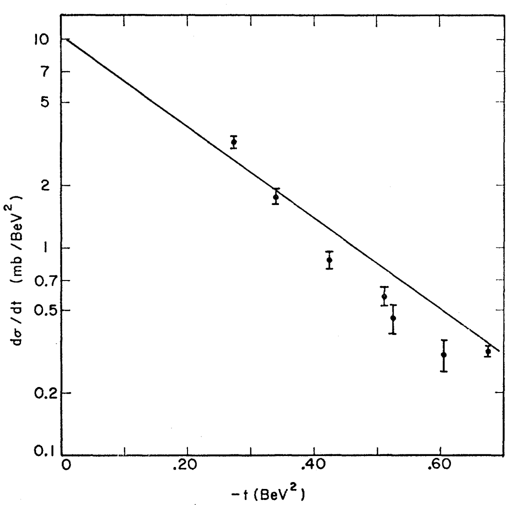

FIG. 3. Fit to $\bar{K}p$ elastic scattering near 10 BeV for small momentum transfer. Experimental points are taken from Ref. 1. Solid line is prediction described in text using dipion with $a=0.60$.

It should also be pointed out that a future high-energy theory which includes moving branch point contributions in the complex J plane (as well as Regge poles) may completely change the pole analysis; the arguments concerning residue functions are applicable only to poles and not to branch-cut discontinuities, and the branch-point contributions may in fact dominate the pole terms.

There may be a theoretical connection between the Pomeranchon pole formula (with vanishing slope for the trajectory) and our formula (4.2) for the peripheral inelastic contributions to elastic scattering. Both give a purely imaginary amplitude in first approximation, a total cross section which is a constant at high energies, and a diffraction pattern which does not change shape with increasing energy. The theoretical bridge could be supplied by considering multiparticle states which contribute to the Pomeranchon pole structure, in some approximation such as the multiperipheral field-theoretic model developed by Amati, Fubini, and Stanghellini.[19] This sort of reasoning has been developed by Feinberg and Chernavskii.[20]

By allowing the (1,1) K-matrix element to be nonzero, we can account for the fact that pp and $\bar{p}p$ elastic scattering amplitudes are not the same, and similarly for the K^-p compared to K^+p amplitudes. However, one-meson pole terms alone cannot account for the difference, since the same difficulties exist in such a model as with the Regge-pole model[17] concerning the signs of the pole residues. Thus, we cannot explain the difference unless we take the K_{11} terms from a model including multiparticle states in the t channel.

If a model for K_{11} is available, it is also possible to obtain a shrinking diffraction pattern for pp scattering without the explicit introduction of Regge poles, by

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[17] W. Rarita and V. L. Teplitz, Phys. Rev. Letters **12**, 206 (1964).
[18] H. Cheng and D. Sharp, Phys. Rev. **132**, 1854 (1963); Sec. III.
[19] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento **26**, 896 (1962).
[20] Section V of E. L. Feinberg and D. S. Chernavskii, Usp. Fiz. Nauk **82**, 3 (1964) [English transl.: Soviet Phys.—Usp. **7**, 1 (1964)].

adjusting the energy dependence of K_{11}. Similar observations have been made by Durand and Greider.[21]

One final observation may be inserted here concerning nucleon-nucleon scattering; the curves for pp and $\bar{p}p$ seem to be most similar for small momentum transfers.[1] This is a favorable indication for a model such as presented here for $\bar{p}p$ scattering, since we expect the peripheral one-pion exchange processes to be quite similar for these two cases even if the interactions differ significantly at smaller b values.

In order to settle on a model for the K-matrix elements in the peripheral region, which can be used in the succeeding sections to unitarize one-meson exchange processes, we adapt the form implied by (4.2); in the notation (2.14) (for $\bar{p}p$ reactions)

$$C_i=(s/2M)^{1/2}a_iK_0(\mu b)\,, \tag{4.4}$$

where $a=\sum_j a_j^2$. A similar form is to be used for $\bar{K}p$ reactions, with μ replaced by 2μ.

V. HIGH-ENERGY $\bar{K}N$ CHARGE EXCHANGE

Experimental data at moderately high energies[22] on the charge exchange reaction $K^-+p\rightarrow \bar{K}_0+n$ indicate a strong forward peak. This leads one to suspect that the process may be dominated by one-meson exchange. The ρ meson is the lightest one which can contribute if selection rules appropriate to strong interactions are taken into account. There is some difficulty with explaining the small dip in the forward direction as indicated by the 1.8 GeV/c[22] data, but we assume that this will disappear if the energy is sufficiently high.

A simple calculation of the magnitude of the cross section for this process from the perturbation-theory diagram for ρ exchange was performed. The coupling constant factor $f_{\rho KK}f_{\rho NN}/4\pi$ is the only adjustable parameter, if we take the ρN anomalous-moment coupling strength (relative to the electric part of the coupling) from the analysis of nucleon electromagnetic form factor data. An estimate of the expected magnitude of this number can be obtained from combining the experimental 2-pion decay width of the ρ with theoretical ratios derived from the idea of universal isospin coupling of the ρ[23] or the octet models in unitary symmetry,[24] the predicted values coincide in these two approaches. We expect on this basis a value of 2.0 for $f_{\rho KK}f_{\rho NN}/4\pi$. On the other hand, fitting the experimental charge cross section at 1.8 GeV/c 0.6 mb/sr near the forward direction[22] with the perturbation-theory amplitude calls for $f_{\rho KK}f_{\rho NN}/4\pi=0.2$. This factor of 10 discrepancy would lead one to believe that there is a serious quantitative disagreement between the one-meson exchange model and experiment, although the width of the forward peak is roughly in agreement with a ρ-exchange model.

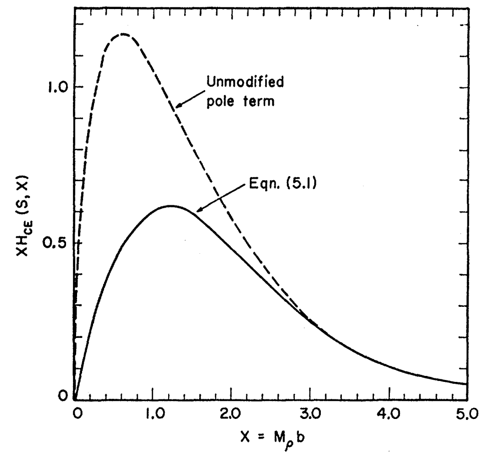

FIG. 4. Solid line gives $xH_{CE}(s,x)$ for $\bar{K}N$ exchange reaction, where $x=m_\rho b$; H_{CE} taken from Eq. (5.11). Unmodified pole term is shown for comparison.

On the basis of our K-matrix approach, we can correct this simple model by explicitly taking into account peripheral inelastic processes. To accomplish this, it is sufficient to assume that the inelastic processes dominate the numerical value of the denominators in (2.15) and (2.16); then we can take the denominators from our fit to $\bar{K}p$ elastic scattering in the previous section.[25] The charge-exchange amplitude is a difference between the eigenamplitudes for $I=1$ and $I=0$ scattering. The resulting Fourier-Bessel component of the amplitude, ignoring spin, will be of the form

$$H_{\rm CE}(s,b^2)=\frac{H_{\rm CE}{}^{(\rho)}(s,b^2)}{1+aK_0{}^2(2\mu b)}\,. \tag{5.1}$$

We have introduced the denominator from Eq. (4.2), with numerical coefficient and meson mass chosen to fit the $\bar{K}p$ elastic-scattering data. The numerator $H_{\rm CE}{}^{(\rho)}$ is the uncorrected perturbation-theory (meson-pole) charge-exchange amplitude in the Fourier-Bessel representation. We ignore the spin-flip amplitude, since the explicit computations above showed it was numerically dominated by the nonspin-flip term when ρ exchange is the initial approximation.

A graph of this function is given in Fig. 4, with the uncorrected Fourier-Bessel amplitude for comparison. The computed cross section, normalized to the data near the forward direction, is shown in Fig. 5. The required value for $f_{\rho KK}f_{\rho NN}/4\pi$ now is 0.7, which is a distinct improvement in bringing theory closer to experiment. It is clear that in general, cross sections for

[21] L. Durand and K. R. Grieder, Phys. Rev. **132**, 1217 (1963).
[22] P. M. Dauber, Phys. Rev. **134**, B1370 (1964).
[23] J. J. Sakurai, Ann. Phys. (N.Y.) **11**, 1 (1960).
[24] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); J. J. Sakurai, in *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963).
[25] The forward diffraction peak in $\bar{K}p$ scattering seems to have the same shape down to 0.2 BeV; R. Crittenden, H. J. Martin, W. Kernan, L. Leipuner, A. C. Li, F. Ayer, L. Marshall, and M. L. Stevenson, Phys. Rev. Letters **12**, 429 (1964).

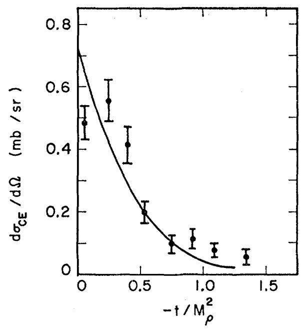

FIG. 5. Fit to $\bar{K}N$ charge exchange cross section for 1.80 BeV/c beam momentum. Data points are from Ref. 22; solid line is prediction from Eq. (5.1) for ρ exchange modified by unitarity, with $f_{\rho NN}f_{\rho KK}/4\pi=0.70$.

processes involving heavy-meson exchanges are suppressed to a considerable extent by the competition of peripheral inelasticity, and angular distributions for such processes are peaked somewhat more sharply in the forward direction than would be expected from the uncorrected pole approximation.

VI. HIGH-ENERGY NEUTRON-PROTON CHARGE EXCHANGE

Experiments at 2.04 and 2.85 BeV have indicated a very sharp forward peak in the np charge-exchange cross section,[26] suggesting a peripheral process induced by one-pion exchange. The perturbation-theory pole contribution for this process was examined by Phillips,[27] who pointed out that this term alone would vanish in the forward direction since it contains t in the numerator. Phillips suggested that constructive interference may take place between the one-pion term and other "background" components of the elastic-scattering amplitude, resulting in a narrow forward peak which could account for the experimental result.

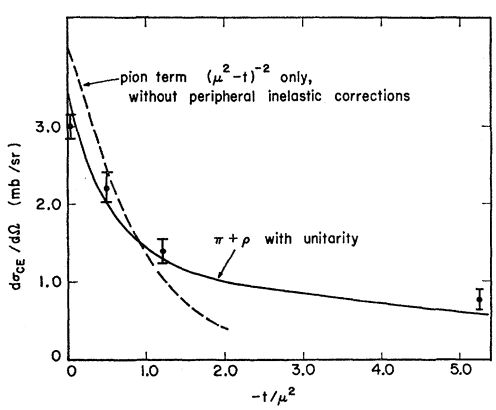

FIG. 6. Fit to neutron-proton charge-exchange cross section at 2.85 BeV. Experimental points are from Ref. 26; solid line is π plus ρ superposition modified by inelastic unitarity, as described in text.

An alternative model was proposed by Muzinich,[28] using only the ρ-meson exchange amplitude, which was formulated as a Regge pole term. By suitable choice of Regge parameters, it was possible to fit the charge-exchange data in Ref. 24. It was subsequently pointed out by Phillips[29] that such a fit does not seem to be consistent with an analysis of the difference between the pp and np total cross section, and that the residue of the ρ Regge pole must change sign near $t=0$ in Muzinich's model.[30] Later, a more complicated model was proposed by Ahmadzadeh[31] who introduced another heavy isovector meson to correct the difficulties with ρ exchange in Muzinich's model.

Our analysis will return to the idea of a basic process caused by one-pion exchange, but corrected through use of the K-matrix formulas for the inelastic competition and diffraction scattering in the peripheral region. We will consider first a spinless model based on a one-pion-exchange matrix element proportional to $t/(\mu^2-t)$, which vanishes in the forward direction, and show how the unitarity modifications change this matrix element to give a forward peak. Then we will consider the spin structure of nucleon-nucleon scattering and show in the actual case how it is possible to obtain the narrow forward peak through inelastic effects. A definite prediction is not possible in the latter case, however, since it is necessary to examine the spin dependence of inelastic channels, for which data are not available.

Beginning with the spinless model, we note that the unitarity saturation of low partial waves as discussed at the end of Sec. II will be quite important here. Deletion of the S-wave (or small b) component allows us to replace $t/(\mu^2-t)$ with $\mu^2/(\mu^2-t)$, which does not vanish in the forward direction; in fact, this already gives approximately the right charge-exchange amplitude for small angles. Denoting the Fourier-Bessel transform of the charge-exchange amplitude based on this replacement by $H_{\mathrm{CE}}{}^{(\pi)}$, we have a formula for H_{CE} identical in form with (5.1)

$$H_{\mathrm{CE}}(s,b^2)=H_{\mathrm{CE}}{}^{(\pi)}(s,b^2)/\Delta(b)\,, \qquad (6.1)$$

where $\Delta(b)$ has the same form as the denominator in (5.1) with a (and μb as the argument of K_0) chosen to fit the proton-proton elastic scattering amplitude. Numerical fits in this case show that (6.1) yields approximately a 50% reduction in cross section in the forward direction compared to the pole term, and an increased peaking toward small angles which brings the theoretical curve into good agreement with experiment. The more slowly varying contributions, which are still appreciable for $(-t)^{1/2}>\mu$, may be ascribed to a ρ-exchange contribution. The fit to data based on (6.1)

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[26] H. Palevsky, J. A. Moore, R. L. Stearns, H. R. Muether, R. J. Sutter, R. E. Chrien, A. P. Jain, and K. Otnes, Phys. Rev. Letters **9**, 509 (1962).
[27] R. J. N. Phillips, Phys. Letters **4**, 19 (1963).
[28] I. J. Muzinich, Phys. Rev. Letters **11**, 88 (1963).
[29] R. J. N. Phillips, Phys. Rev. Letters **11**, 442 (1963).
[30] The undesirability of such a situation has been discussed in Sec. IV.
[31] A. Ahmadzadeh, Phys. Rev. **134**, B633 (1963).

plus a ρ contribution of the same form is given in Fig. 6, with the function $(\mu^2-t)^{-2}$ for comparison.

We observe that the suppression of the magnitude of the off-diagonal peripheral cross section is less in this example than in the ρ exchange case treated in Sec. V. The physical reason is clear; in the pion-exchange case, the effective range of interaction is comparable to or larger than that of the inelastic process, leaving much of the function $H(b)$ undisturbed; whereas in the heavy-meson processes the inelastic terms have a larger range and suppress a great deal more of the pole term's b components.

A similar pole-term model has been investigated by Islam and Preist[32]; they are able to fit the data moderately well, but utilize form factors to obtain the required behavior for the pion term. Since their pion form factors are just those required phenomenologically to account for other experiments involving one-pion exchange,[3] it is not surprising that our approaches agree in the end. The ρN coupling constant they require, $f_{\rho NN}^2/4\pi=0.4$, is rather small compared to universality predictions[24,25]; our fit allows a larger value (about 1.0) because of the unitarity effects.

We now turn to the realistic case including nucleon spin.[33] The helicity formulation of nucleon-nucleon scattering appropriate for relativistic energies has been presented by Goldberger, Grisaru, MacDowell, and Wong (GGMW).[34] Combining the formulas (6.6) of GGMW with (3.5) of Ref. 35, we find the one-pion exchange forms of the $I=1$ partial wave amplitudes h^J [defined in formula (3.1) of Ref. 35] as follows:

$$\begin{aligned} h_{11}{}^J=-h_0{}^J&=-\frac{1}{4}\frac{g^2}{4\pi}\left[Q_J(X_0)-\frac{JQ_{J-1}(X_0)+(J+1)Q_{J+1}(X_0)}{2J+1}\right],\\ h_1{}^J=-h_{22}{}^J&=-\frac{1}{4}\frac{g^2}{4\pi}\left[Q_J(X_0)-\frac{(J+1)Q_{J-1}(X_0)+JQ_{J+1}(X_0)}{2J+1}\right],\\ h_{12}{}^J&=0, \end{aligned} \tag{6.2}$$

where $X_0=1+\mu^2/2k^2$. The $I=0$ amplitudes are (-3) times the $I=1$ amplitudes. The charge-exchange amplitude is the difference between $I=0$ and $I=1$ amplitudes *after* corrections from inelastic unitarity have been included.

The unitarity relations now involve the uncoupled $h_0{}^J$ and $h_1{}^J$ amplitudes, and the triplet amplitudes $h_{11}{}^J$, $h_{12}{}^J$, $h_{22}{}^J$ are coupled among themselves.[34]

We assume that those helicity amplitudes which are zero before corrections are applied (one-pion exchange only) remain zero when unitarity is enforced. Then we have only the two amplitudes ϕ_2 and ϕ_4, which have the angular momentum decomposition[34,35]

$$\begin{aligned} \phi_2(s,t)&=E^{-1}\sum_J(2J+1)d_{00}{}^J(z)[h_0{}^J(s)-h_{11}{}^J(s)],\\ \phi_4(s,t)&=E^{-1}\sum_J(2J+1)d_{1,-1}{}^J(z)[h_{22}{}^J(s)-h_1{}^J(s)]. \end{aligned} \tag{6.3}$$

These may be converted to the impact-parameter representation by use of the asymptotic forms (for large J) of the Legendre functions; after utilizing formula (3.30) of Ref. 35, and passing to the limit of a large number of angular momentum contributions, we obtain the representations

$$\begin{aligned} \phi_2(s,t)&=(2k^2/E)\int_0^\infty bdb[h_0(b,s)-h_{11}(b,s)]J_0(b(-t)^{1/2}),\\ \phi_4(s,t)&=(2k^2/E)\int_0^\infty bdb[h_{22}(b,s)-h_1(b,s)]\\ &\quad\times\left[J_0(b(-t)^{1/2})-(1+z)\times\frac{J_1(b(-t)^{1/2})}{b(-t)^{1/2}}-J_2(b(-t)^{1/2})\right]. \end{aligned} \tag{6.4}$$

The functions $h(b,s)$ here are to be identified with $h^J(s)$ for large J, where $kb=J+\frac{1}{2}$. We have removed any terms which are singular for small J values, as required by unitarity. The total charge-exchange cross section will be proportional to the sum of the squares of ϕ_2 and ϕ_4.

Note that ϕ_2 has an angular dependence identical to that of the spinless model, whereas ϕ_4 will always vanish in the forward direction. Thus, if we are to ascribe the sharp forward peak to the pion-exchange process modified by inelastic unitarity, it seems to be necessary that $(h_{22}-h_1)$ be much smaller than (h_0-h_{11}).

Such a situation will obtain if the long-range inelastic processes in the triplet amplitudes (both $J=l$ and $J=l\pm1$) are much stronger than in the singlet (h_0) amplitudes; then the values of h_1 and h_{22} will be damped strongly from the inelastic competition, while h_0 will survive alone to produce the forward peak from ϕ_2. Detailed experimental data on inelastic processes would be necessary, then, for a direct check on this analysis.

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[33] I am indebted to Dr. R. J. N. Phillips for a private communication concerning the fallacies of a spinless treatment.

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VII. PERIPHERAL ISOBAR PRODUCTION MODELS

Some inelastic reactions yielding isobars at high energies, for example $K^+ + p \to K^* + p$ at 3 BeV/c,[36] show a very strong forward peaking in the over-all center-of-mass reaction angle, and appear to be good candidates for one-meson exchange process. There seems to be a considerable variation in the degree with which data for such processes fit simple pole approximations as far as the momentum-transfer dependence is concerned. This has been interpreted by most investigators[2] as evidence that vertex corrections in the field-theoretic sense are very important. However, we have seen in our analysis how the momentum-transfer characteristics may be quite different from the pole approximations in a unitarized theory including inelastic channels. As discussed in the introduction, such effects have been treated in Refs. 4 and 5 by introducing a phenomenological absorptive potential. The physical picture presented by the K-matrix formalism is somewhat more transparent. We will now show how qualitative agreement of most of the available peripheral isobar-production data with a theory based on one-meson pole terms may be achieved. Detailed calculations will depend on the spin structure of the amplitudes, and will not be carried out here.

In our approximate form of the K-matrix approach, every off-diagonal (production) matrix element takes on the form (2.16). This simple form is essentially due to ignoring the interactions of the bodies in the final states.

It is apparent that predictions of the one-meson pole terms concerning the alignment or relative population of spin states of the isobars are not affected by the peripheral inelastic damping corrections, which appear in the denominator of (2.16). As a consequence, predictions such as the Stodolsky-Sakurai[12,37] vector-meson model makes concerning final-state (isobar decay) angular correlations remain valid in our K-matrix model. These predicted correlations are consistent, in every case which is probably peripheral, with the experimental[33,38] data. On the other hand, we expect the inelastic unitarity corrections will increase the forward peaking of all such reactions; much more so in cases where the elastic scattering of the initial-state particles exhibits a diffraction character, with a large total cross section, signifying strong inelastic peripheral processes.

This qualitative consideration appears to fit quite well with the reactions considered in Refs. 2 and 3. In comparing the theoretical curves given in those references, we must first remember to subtract out in the matrix element any terms which go to a constant for large t as explained at the end of Sec. II before squaring to obtain the cross section. This immediately clears up the main difficulty with the curves given in Ref. 2, Fig. 3, which treat the reaction $\pi^+ + p \to \omega^0 + N_{3/2}^*$. The matrix elements in the one-meson exchange model have an additional power of t in the numerator due to the large total spin of the final states, compared to reactions such as $\pi^+ + p \to \pi^0 + N^*$. The latter reaction (Fig. 4 of Ref. 2) can be fit correctly after unitarity corrections are put in. Note that these two processes, with $\pi^+ p$ initial states, deviate more strongly from the pole terms than $K^+ + p \to K^0 + N_{3/2}^*$ (Fig. 2 of Ref. 2); the latter is already quite well fit. We expect this trend, since $K^+ p$ reactions have a smaller total cross section and smaller elastic cross section than $\pi^+ p$ reactions at the energies under consideration. Furthermore, we predict that $\bar{K} p$ reactions will show about the same degree of forward enhancement as the $\pi^+ p$ reactions as long as the $\bar{K} p$ and $\pi^{\pm} - p$ cross sections are comparable at the same energies; around 3 BeV this is the case. Some caution must be used, however, in predictions for energies below 3 BeV, since the $\pi^+ p$ and $\pi^- p$ diffraction peaks do not exhibit identical behavior for these low energies.[39]

ACKNOWLEDGMENT

The numerical transforms involved in Eq. (2.2) were carried out on the IBM 7094 at the UCLA computing facility.

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[38] B. Kehoe, Phys. Rev. Letters **11**, 93 (1963); J. Bartsch *et al.*, Phys. Letters **10**, 229 (1964); G. B. Chadwick *et al.*, Phys. Letters **6**, 309 (1963).

[39] D. E. Damouth, L. W. Jones, and M. L. Perl, Phys. Letters **11**, 287 (1963).